

SEASONAL VARIATION OF ANGULAR MOMENTUM TRANSPORT AT 500 MB.

A. J. MILLER, S. TEWELES, and H. M. WOOLF

Weather Bureau, ESSA, Silver Spring, Md.

ABSTRACT

Monthly mean values of the geostrophic angular momentum transport at 500 mb. have been computed as a function of latitude and zonal wave number (1 through 10) for a 10-yr. period.

The total transport is found to be in good agreement with previous calculations; at the same time several wave numbers exhibit considerable individuality. Equatorward transport by wave 2 at high latitudes extends farther south, and is much larger in magnitude, than the transport by any of the other wave numbers. Also, the negative transport in low latitudes is in distinct contrast to the behavior of the other waves. Wave 3, on the other hand, transports momentum poleward in mid-latitudes at a rate at least twice as great as that of any other wave number.

An additional finding is that in July and August, waves 1 through 5 are relatively inactive in transporting momentum, while waves 6 through 10 accomplish substantial transport near the latitude of the summertime maximum westerlies.

1. INTRODUCTION

It has long been realized that the large-scale, zonally asymmetric disturbances in the atmosphere generally transport relative angular momentum from the low- and high-latitude easterly wind regimes into the mid-latitude westerly current. Starr [9] and Mintz [3], utilizing observed and geostrophic wind data respectively, were among the first to verify this hypothesis of Jeffreys [1], while Saltzman [4] and van Mieghem et al. [12] extended the analysis to the wave number domain.

In recent years, extensive studies, exemplified by those of Saltzman and Fleisher [5, 6], Saltzman and Teweles [7], Wiin-Nielsen et al. [13] and Krueger et al. [2], have demonstrated that the quasi-horizontal "eddies" play a major role in maintaining the kinetic energy of the zonal flow against frictional dissipation. The foundation of these studies, however, is the latitudinal distribution of the angular momentum transport and mean zonal flow. Hence, it is of concern that to date no clear picture has emerged that depicts this momentum transfer as a function of wave number in the time-mean sense.

It is our purpose to present a 10-yr. monthly average of the 500-mb. angular momentum transport in the Northern Hemisphere as a function of latitude and wave number, and to relate this transport to the seasonal and latitudinal variations of the mean zonal wind.

2. PROCEDURE

The meridional flux of relative angular momentum across a latitude wall, per unit time, is given by

$$ag^{-1} \cos \phi \int_0^{p_0} \int_0^{2\pi} a \cos \phi u v d\lambda dp \quad (1)$$

where

a =distance from the center of the earth

ϕ =latitude

λ =longitude

p =pressure

u =eastward component of the wind

v =northward component of the wind

g =acceleration due to gravity.

The flux per unit pressure difference is then given by

$$ag^{-1} \cos \phi \int_0^{2\pi} a \cos \phi u v d\lambda. \quad (2)$$

If we now expand u and v in terms of zonal harmonics, where n is wave number,

$$u = \sum_{n=-\infty}^{\infty} U(n) e^{in\lambda} \quad v = \sum_{n=-\infty}^{\infty} V(n) e^{in\lambda}$$

then (2) becomes:

$$\begin{aligned} a^2 g^{-1} \cos^2 \phi \int_0^{2\pi} \sum_{n=-\infty}^{\infty} U(n) e^{in\lambda} \sum_{m=-\infty}^{\infty} V(m) e^{im\lambda} d\lambda \\ = 2\pi a^2 g^{-1} \cos^2 \phi \sum_{n=-\infty}^{\infty} U(n) V(-n) \end{aligned} \quad (3)$$

and the flux of relative angular momentum by the "eddies" ($n \neq 0$) per unit time per unit pressure difference is

$$T_m(\phi) = 2\pi a^2 g^{-1} \cos^2 \phi \sum_{n=1}^{\infty} \{ U(n)V(-n) + U(-n)V(n) \}. \quad (4)$$

The daily 1200 GMT Northern Hemisphere 500-mb. analyses of the National Meteorological Center, for the 10-yr. period April 1, 1955 through March 31, 1965, were subjected to zonal harmonic analysis. Harmonic coefficients of the geostrophic wind components were determined, and angular momentum transports computed in the wave number domain. Daily values of these quantities were obtained for wave numbers 1 through 15 at every

5° of latitude from 17.5° N. to 77.5° N., and averaged to obtain monthly means. The latter quantities were then averaged over the 10-yr. period.

Results of these computations are given for the longitudinally averaged zonal wind ($n=0$) and for the relative angular momentum transports by waves 1 through 10. The results are presented mainly in tabular form,¹ but as an aid in interpretation the transports by wave numbers 1, 2, 3, 6, and the sum of waves 1 through 10 are also

¹ Individual values are truncated.

TABLE 1.—Ten-year annual mean of zonal wind ($n=0$) and relative angular momentum transport ($n=1-10$)

		Latitude (° N.)												
		17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
		Zonal Wind (m./sec.)												
(n=0)-----		2.6	6.2	9.5	11.0	13.9	14.3	12.7	9.8	7.5	5.9	5.3	4.9	3.5
Wave No.		Relative Angular Momentum Transport (10^{21} ergs(mb.)												
$n=1$ -----		0	4	10	16	22	21	15	9	2	-1	-1	-1	-0
2-----		-10	7	13	13	11	0	-10	-18	-26	-23	-16	-6	-0
3-----		13	24	43	65	72	64	38	16	1	-5	-4	-1	-0
4-----		2	10	20	28	25	18	11	1	-3	-3	-2	-0	0
5-----		1	8	18	27	28	23	12	3	-0	-1	-0	-0	0
6-----		6	13	24	30	24	19	15	6	0	-1	-0	0	0
7-----		8	14	22	24	21	16	10	3	0	-0	-0	-0	0
8-----		3	6	12	18	17	14	8	2	-0	-0	-0	-0	0
9-----		4	6	8	12	11	8	5	0	-0	-0	-0	-0	0
10-----		2	5	6	7	6	4	2	0	-0	-0	-0	0	0
Σ -----		32	100	180	245	241	190	109	26	-27	-37	-25	-8	-1
$n=1$ -----														
Σ -----		37	109	192	259	250	196	111	26	-27	-38	-25	-9	-3

TABLE 2.—Ten-year mean values and 95 percent confidence limits of zonal wind. Units: m. sec.⁻¹

		LATITUDE (°N.)												
		17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
Mo.		MEANS												
J	8.6	15.3	19.2	19.8	18.4	15.8	12.5	9.4	7.5	6.1	5.2	4.4	3.3	
F	9.8	15.6	19.5	20.2	18.8	15.8	11.5	8.2	6.8	5.7	5.3	5.0	4.1	
M	7.5	13.3	17.4	19.0	18.4	15.8	11.8	8.3	7.0	6.5	6.4	5.7	3.8	
A	5.5	9.0	12.3	14.2	15.1	14.6	12.5	9.7	7.6	6.3	5.8	5.1	3.1	
M	2.0	5.2	8.4	10.8	12.4	12.8	11.5	8.7	6.4	5.1	4.8	4.5	3.4	
J	-1.6	+0.9	4.5	8.0	10.8	11.4	10.0	7.4	5.2	4.0	4.4	4.4	3.2	
J	-2.7	-2.3	-0.2	3.4	7.9	10.8	10.7	7.5	4.7	3.4	4.4	5.6	5.0	
A	-3.0	-2.3	-0.6	2.5	7.2	11.0	12.0	9.8	6.6	4.5	4.6	4.4	2.6	
S	-3.0	-1.8	+0.9	5.4	10.1	13.1	13.5	11.3	8.8	6.8	5.9	5.1	3.5	
O	-0.1	2.5	5.8	9.1	12.2	14.7	15.3	13.5	10.8	8.1	6.5	5.8	4.1	
N	2.4	6.9	11.2	13.9	16.4	17.2	15.6	12.2	9.3	7.0	5.3	4.1	2.7	
D	5.9	11.2	15.6	17.8	18.5	17.7	15.0	11.4	8.6	6.3	4.4	3.4	3.1	
		CONFIDENCE LIMITS												
J	2.4	0.8	1.0	1.1	1.2	1.5	1.6	1.7	1.4	1.4	1.7	1.7	1.8	
F	1.3	1.1	1.4	1.3	0.9	0.8	1.0	1.1	1.2	1.3	1.3	1.8	2.2	
M	1.2	1.2	1.5	0.8	0.5	1.0	1.4	1.3	1.6	2.0	1.6	1.7	2.4	
A	0.5	0.7	0.8	0.8	0.9	0.7	0.9	1.0	0.9	0.8	0.7	1.1	1.8	
M	1.1	1.0	0.8	0.8	0.9	0.8	1.1	1.2	1.0	1.1	1.3	1.3	1.4	
J	1.2	0.6	0.7	0.7	0.6	0.5	0.8	1.0	0.8	0.8	0.6	0.6	1.7	
J	1.3	0.5	0.5	0.4	0.5	0.7	0.5	0.8	1.0	1.0	1.0	1.1	1.3	
A	1.2	0.5	0.5	0.4	0.6	0.9	0.9	1.0	1.1	1.0	0.9	1.4	1.6	
S	0.8	0.6	0.9	0.7	0.6	0.5	0.8	0.9	0.6	0.6	1.1	1.1	1.4	
O	0.9	1.0	1.1	0.9	1.0	1.1	0.9	0.8	1.1	1.2	1.0	0.8	1.0	
N	0.5	0.6	0.6	0.9	0.7	1.0	0.9	1.0	1.0	1.2	1.3	1.1	1.4	
D	1.1	0.5	0.7	1.0	1.2	1.3	1.3	1.4	1.3	1.3	1.1	1.4	1.9	

presented graphically. The truncation at wave 10 was determined by the fact that little additional information is available at the higher wave numbers; and furthermore, the physical significance of the shorter waves appears to be limited to that of a shape parameter. As a measure of reliability, 95 percent confidence limits for the 10-yr. monthly mean values were determined by the Student *t* test.

3. RESULTS

Table 1 presents the annual means, averaged over the 10 years, of the zonal wind ($n=0$) and of the angular momentum transports for each of the wave numbers 1 through 10. It is clear that the flux of angular momentum is generally positive (poleward) from 17.5° N. into mid-latitudes, changing sign at the higher latitudes. The resulting convergence of angular momentum transport ($-\Delta T_m/\Delta\phi$) generally attains maximum strength in the region of maximum zonal wind. Furthermore, wave numbers 2 and 3 behave in a rather anomalous manner as compared to the others. The negative transport by wave 2 at higher latitudes extends farther south, and is much larger in magnitude, than the flux due to any of the others. Also, the negative transport in low latitudes is in distinct contrast to the other waves. Wave 3, on the other hand, transports momentum poleward in mid-latitudes at a rate at least twice as great as that of any other wave number. Comparison of the transports summed over wave numbers 1–10 and 1–15 confirms our earlier remarks concerning the contribution of the higher wave numbers.

The pattern of longitudinally averaged zonal wind (table 2) displays a 15° shift in latitude of the maximum during the course of a year, with the maximum at its lowest latitudes in winter. In conjunction with this seasonal variation, low-latitude easterlies appear in summer up to about 27.5° N.

The maximum poleward momentum transport in wave number 1 (table 3, fig. 1) occurs near 37.5° N. in winter, while secondary positive maxima appear at higher latitudes in June and September. A small negative (equatorward) transport is found in most months at higher latitudes, extending northward to about 37.5° N. It is interesting to note that the summertime easterly zonal flow is generally accompanied by equatorward momentum flux by wave number 1 at low latitudes. The maximum positive transport apparently takes place in latitudes north of the zonal westerly wind maximum in June, September, and the winter months, but in view of the uncertainty in the positions of the momentum flux and wind maxima, the latter relationship may not be significant.

In contrast to wave 1 and, indeed, to all the other waves presented here, wave 2 (table 4, fig. 2) is characterized by large negative transports with their maximum at high latitudes, extending in winter into middle latitudes. To a much greater extent than any other wave, wave 2 exhibits

low-latitude negative fluxes of substantial magnitude, with maximum poleward penetration in summer.

The most notable feature of wave 3 (table 5, fig. 3) is its dominant role in the positive transport in middle latitudes during the cooler months. Transport by this wave number in July and August, on the other hand, is reduced considerably. As with most other waves, the exact location of maximum flux and its relationship to the zonal-wind maximum are not clearly defined. However, convergence of angular momentum transport ($-\Delta T_m/\Delta\phi$) is generally associated with the region northward of the maximum westerlies.

Tables 6–12 indicate that for wave numbers greater than 3 the overall patterns are not sufficiently different from one another to merit detailed, individual attention. In general, they display a latitudinal shift of maximum transport with season in conjunction with the shift of the zonal-wind maximum. The negative transports at high latitudes also show a seasonal displacement with their lowest latitudinal penetration in winter. Within these similar large-scale patterns, however, there are certain distinct features worthy of mention.

In wave 4 (table 6), the positive flux in low and middle latitudes dominates the picture, with the maximum in January at about 32.5° N. Negative flux is found generally north of 52.5° N. except in May when it extends farther south.

As a group, waves 5 (table 7), 6 (table 8, fig. 4), and 7 (table 9) exhibit several maxima in poleward momentum transport during the year. Waves 5 and 7 display two maxima, in winter and spring, while wave 6 has these two plus an additional one in the fall. Furthermore, it is important to note that the summer positive transports for waves 6 and 7 and also waves 8 through 10 (tables 10–12) maintain their strength from the winter season to a larger degree than the planetary waves. In accord with our previous statements concerning the contributions of the higher wave numbers, however, we see a general decrease in flux strength for waves 8, 9, and 10.

Table 13 and figure 5 present the sum of the transports by waves 1 through 10. As might be expected, its overall pattern tends to be smoother than those of the individual waves, but there is a remarkable latitudinal shift of maximum transport with season in conjunction with the shift of the zonal-wind maximum. The maximum holds at 32.5° N. for the six months December–May, but shifts suddenly during June to a July–August position at 47.5° N. from which it slowly retreats southward in the autumn. The negative transports at high latitudes also show a seasonal displacement with their lowest latitudinal penetration in winter.

4. DISCUSSION

We have previously noted that the normal variations in our measured values result in confidence limits for the

TABLE 3.—Ten-year mean values and 95 percent confidence limits of angular momentum transport by wave number one. Units: 10^{21} ergs mb. $^{-1}$

	Mo.	LATITUDE ($^{\circ}$ N.)												
		17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
MEANS														
J	6	19	43	64	76	63	38	18	6	4	3	+0	-1	-1
F	-7	3	24	40	45	38	27	23	14	5	-0	-1	-0	-0
M	-6	2	8	16	17	9	3	-3	-9	-6	-1	-1	-0	-0
A	5	7	7	4	3	1	-2	-5	-7	-7	-6	-3	-1	-1
M	5	+0	-1	+0	5	7	7	6	+0	-4	-3	-2	-0	-0
J	-5	-5	-3	4	17	25	25	21	13	5	1	+0	-0	-0
J	-13	-4	-4	-6	+0	5	6	5	6	+0	-3	-4	-2	-2
A	-3	1	-1	-5	-2	+0	+0	-0	-0	-2	-5	-4	-1	-1
S	3	-0	-2	-0	5	13	18	19	12	3	-1	-2	-0	-0
O	6	3	3	5	9	11	7	5	-3	-8	-7	-4	-1	-1
N	5	11	17	21	31	33	27	9	-5	-2	4	4	1	1
D	7	14	29	45	58	48	30	15	2	-1	-2	-1	-0	-0
CONFIDENCE LIMITS														
J	16	16	18	23	26	27	24	23	15	12	9	5	2	2
F	17	22	32	35	35	36	33	24	20	13	8	7	3	3
M	6	10	10	12	12	12	11	12	11	8	7	5	2	2
A	7	4	4	6	11	16	14	8	5	7	7	6	3	3
M	4	3	5	4	3	8	10	9	8	6	5	4	2	2
J	8	6	5	6	10	12	11	8	9	7	4	3	2	2
J	8	5	6	5	4	6	7	7	6	4	3	3	2	2
A	9	5	4	5	4	3	3	2	2	3	3	3	2	2
S	4	2	1	2	3	5	8	10	9	5	4	4	2	2
O	5	2	2	4	6	8	12	14	15	11	8	4	2	2
N	4	5	8	11	13	19	26	26	23	17	9	5	4	1
D	8	11	13	15	16	21	24	20	14	8	5	5	3	3

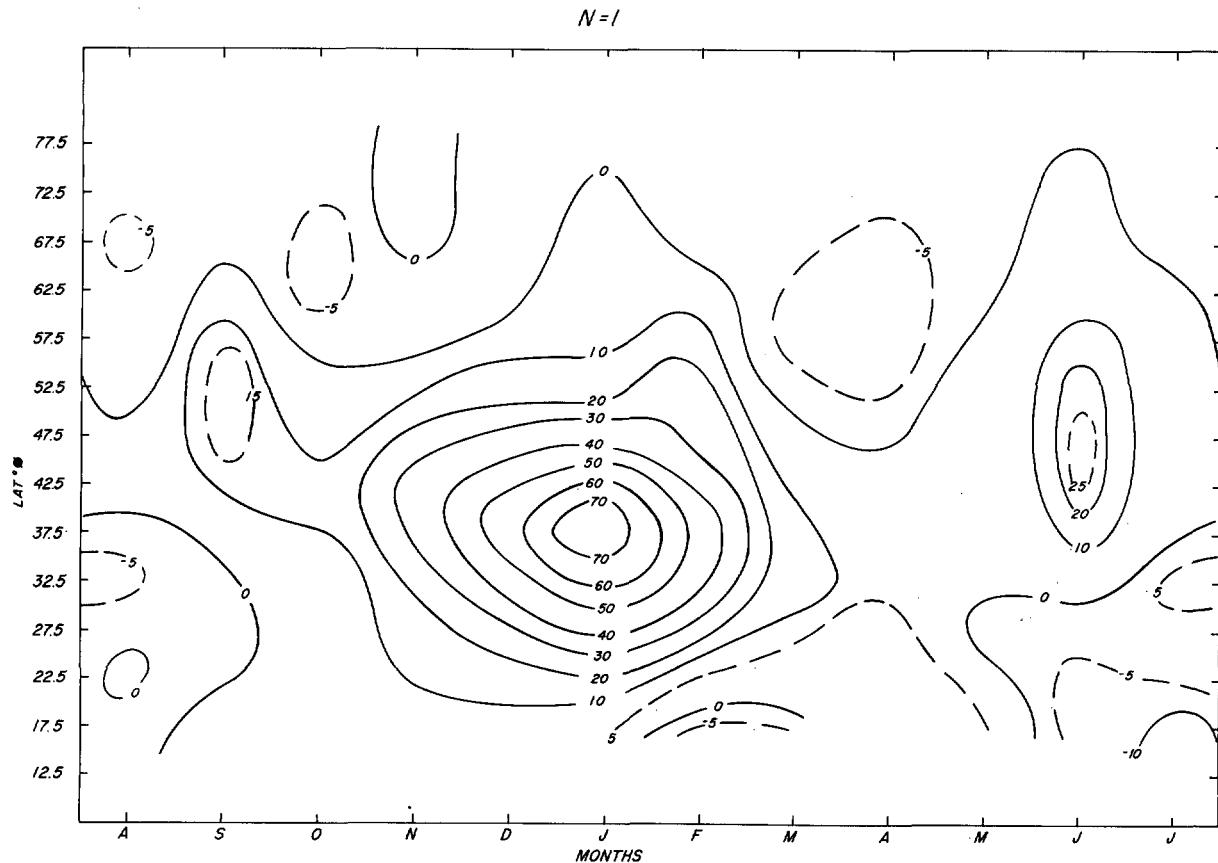


FIGURE 1.—Ten-year average of relative angular momentum transport (10^{21} ergs mb. $^{-1}$) by wave number 1 as a function of latitude and month.

TABLE 4.—Ten-year mean values and 95 percent confidence limits of angular momentum transport by wave number two. Units: 10^{21} ergs mb. $^{-1}$

Mo.	LATITUDE (°N.)												
	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
	MEANS												
J	-15	7	4	+0	-13	-36	-54	-62	-66	-50	-34	-16	-2
F	-14	19	30	43	33	-5	-30	-43	-47	-37	-22	-8	-2
M	3	29	33	35	37	21	-4	-32	-44	-28	-17	-3	1
A	-0	13	18	13	13	10	6	-5	-24	-28	-19	-6	-1
M	2	12	14	11	9	2	-1	-7	-23	-30	-21	-7	-0
J	-13	11	16	17	18	5	-6	-7	-9	-12	-11	-4	-0
S	-32	-11	2	4	4	4	4	3	-0	-1	-1	+0	+0
A	-26	-12	-3	-0	3	4	4	5	+0	-3	-2	-0	+0
S	-19	-12	-1	+0	2	-0	-2	-5	-10	-11	-7	-2	+0
O	-12	-0	7	8	7	6	3	-3	-13	-15	-12	-6	-2
N	-3	16	18	13	11	5	-5	-17	-27	-26	-22	-8	-0
D	2	18	15	9	4	-15	-32	-42	-47	-32	-22	-7	-0
CONFIDENCE LIMITS													
J	13	15	21	28	23	32	40	42	36	30	21	11	4
F	22	34	41	52	62	59	57	46	25	15	11	6	2
M	20	16	19	20	28	31	28	28	21	13	11	6	1
A	8	8	15	16	16	20	16	13	11	10	11	6	2
M	14	12	14	12	18	25	24	20	21	17	12	6	2
J	13	12	11	12	9	10	13	12	9	8	8	4	1
J	20	12	6	9	8	6	7	7	6	6	3	2	1
A	20	13	6	5	5	4	5	5	7	6	6	3	1
S	6	6	8	6	6	8	10	12	11	13	11	6	2
O	8	4	7	8	8	12	14	15	16	13	8	5	2
N	10	15	17	17	18	21	17	15	21	22	14	6	3
D	13	18	32	38	41	38	44	42	24	18	14	7	2

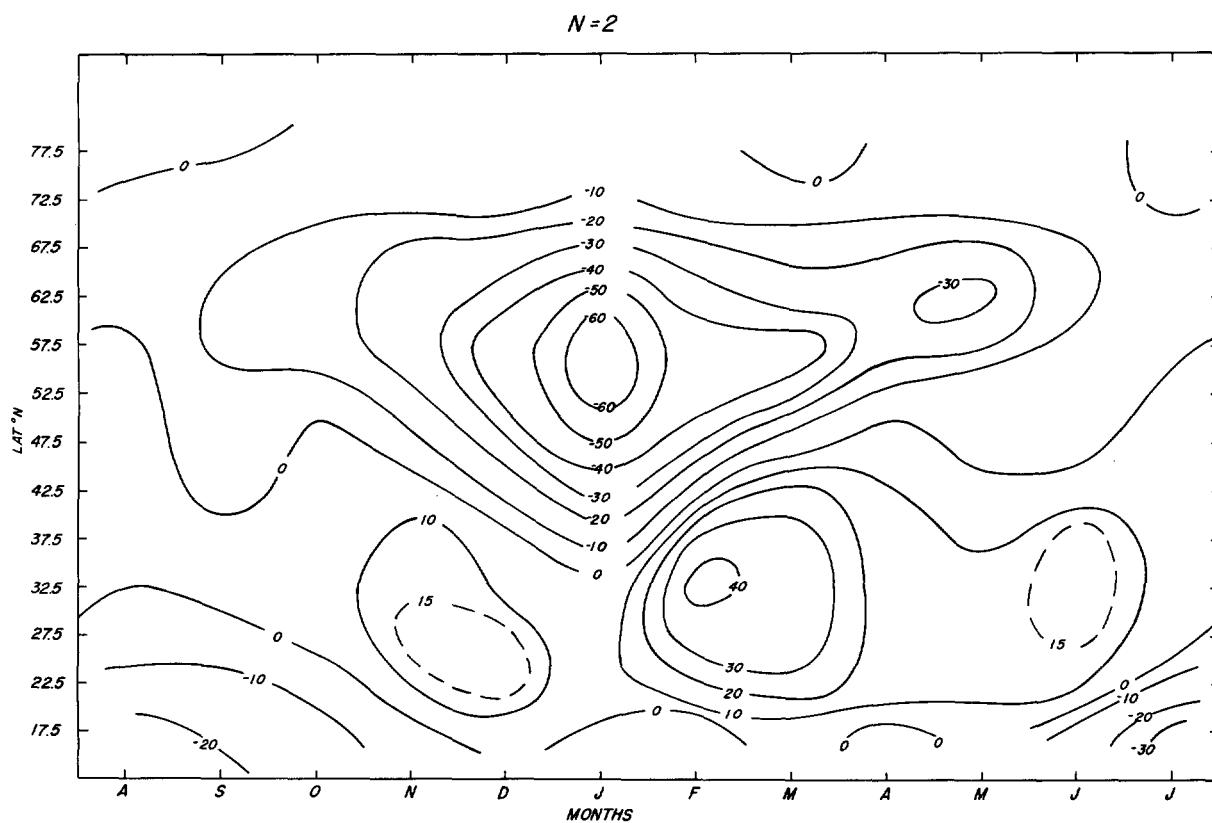


FIGURE 2.—Same as figure 1 for wave number 2.

TABLE 5.—Ten-year mean values and 95 percent confidence limits of angular momentum transport by wave number three. Units: 10^{21} ergs mb. $^{-1}$

Mo.	LATITUDE (°N.)												
	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
MEANS													
J	10	40	81	133	155	139	75	37	8	-10	-10	-3	+0
F	33	64	112	169	184	161	72	14	-3	-12	-9	-1	-0
M	26	46	83	126	146	120	58	13	-3	-11	-6	-1	-0
A	14	21	49	65	67	57	35	8	-0	-1	+0	1	+0
M	11	18	33	41	38	33	29	18	-3	-1	-1	-0	+0
J	21	21	20	20	25	30	21	1	-8	-8	-5	-2	-0
J	4	-0	-0	-1	3	6	8	6	1	-1	-1	-0	+0
A	-0	-0	2	3	4	8	12	15	7	-1	-4	-1	-0
S	4	5	7	12	17	22	22	16	4	-4	-7	-3	-0
O	9	12	20	35	33	28	27	19	10	5	2	1	+0
N	9	19	41	63	71	67	47	23	-1	-8	-3	-1	-0
D	13	38	75	116	118	93	46	18	-0	-7	-2	+0	+0
CONFIDENCE LIMITS													
J	27	21	45	65	72	79	61	41	23	15	11	3	2
F	25	29	51	70	82	78	64	54	32	17	6	3	1
M	19	30	43	50	47	46	45	33	21	10	5	2	0
A	11	13	16	22	31	34	27	17	12	10	6	3	1
M	16	9	9	10	13	16	18	13	7	5	3	2	0
J	10	7	12	13	12	15	14	15	15	9	6	2	0
J	10	4	4	5	3	6	10	9	9	6	5	2	0
A	5	3	2	3	4	6	11	9	6	6	4	2	0
S	6	4	4	6	8	15	20	21	18	10	4	2	0
O	9	9	13	18	25	32	33	30	23	13	6	2	0
N	9	7	11	17	22	31	32	31	24	11	6	3	0
D	23	16	23	35	39	41	43	26	16	11	10	5	0

N=3

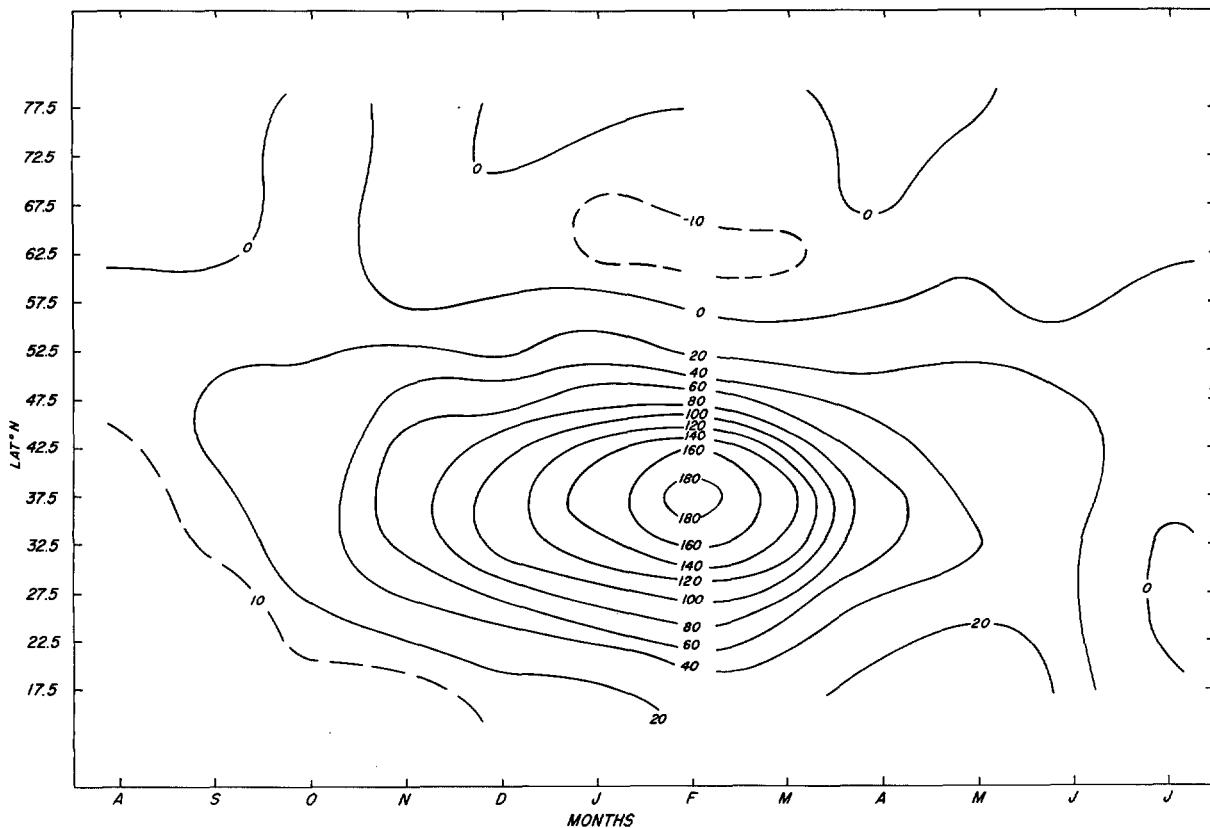


FIGURE 3.—Same as figure 1 for wave number 3.

TABLE 6.—Ten-year mean values and 95 percent confidence limits of angular momentum transport by wave number four. Units: 10^{21} ergs mb. $^{-1}$

Mo.	LATITUDE ($^{\circ}$ N.)												
	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
	MEANS												
J	5	22	43	62	58	45	31	11	-1	-2	-2	-1	-0
F	4	12	36	55	53	40	23	4	-11	-8	-1	+0	+0
M	+0	21	43	52	49	39	18	12	5	-3	-3	-0	+0
A	-1	11	27	38	37	22	6	-16	-20	-16	-2	+0	+0
M	6	14	27	34	16	-4	-12	-20	-16	-7	-2	+0	+0
J	1	4	5	7	4	2	3	-3	-7	-4	-2	1	+0
J	3	5	4	2	-3	-4	+0	5	6	-0	-1	+0	+0
A	1	4	3	2	+0	1	8	11	8	+0	-4	-1	-0
S	1	5	8	5	2	-0	2	+0	2	+0	-1	-0	-0
O	-0	1	5	11	8	8	15	9	3	3	1	1	+0
N	1	4	16	30	23	14	11	-2	-1	-1	-2	-0	+0
D	6	11	26	42	57	50	30	4	-4	-5	-3	-0	+0
CONFIDENCE LIMITS													
J	12	13	20	26	26	23	20	18	17	13	6	2	0
F	13	11	15	25	24	32	38	29	18	9	2	1	0
M	10	13	21	20	22	30	28	19	12	6	5	1	0
A	8	4	7	10	17	26	32	28	18	10	3	1	0
M	7	6	11	17	17	18	12	8	9	7	3	0	0
J	8	4	4	5	8	9	10	10	10	7	3	2	0
J	8	6	7	6	6	6	5	7	7	5	2	0	0
A	8	3	4	4	5	7	11	11	14	16	9	1	0
S	8	6	7	7	6	9	11	14	27	20	11	5	0
O	8	7	10	16	19	19	23	23	27	20	10	7	0
N	10	7	16	18	20	23	25	25	22	10	7	5	1
D	13	17	17	23	25	25	24	19	12	4	2	1	0

TABLE 7.—Ten-year mean values and 95 percent confidence limits of angular momentum transport by wave number five. Units: 10^{21} ergs mb. $^{-1}$

Mo.	LATITUDE ($^{\circ}$ N.)												
	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
	MEANS												
J	3	18	37	56	68	39	10	-0	-2	-1	-0	+0	+0
F	+0	11	26	33	24	16	8	+0	-4	-4	-1	+0	+0
M	2	12	23	30	22	17	13	2	-4	-2	-0	-0	+0
A	8	19	39	59	48	33	23	13	3	-2	-1	-0	+0
M	-1	4	16	21	19	18	10	3	3	-0	-1	-0	-0
J	-0	2	5	7	13	13	8	6	-0	-1	-0	-0	+0
J	+0	1	1	2	-2	-0	4	8	3	-4	-4	-1	-0
A	+0	1	-1	2	5	5	2	-1	-4	-4	-1	-0	+0
S	4	3	2	8	12	15	15	8	3	2	+0	+0	+0
O	3	3	13	18	23	21	9	-6	-4	-1	-1	-0	-0
N	5	12	24	43	50	50	32	16	1	-2	-0	+0	+0
D	-4	11	29	49	59	44	10	-4	-5	-2	-0	-0	-0
CONFIDENCE LIMITS													
J	16	15	22	38	40	45	32	17	8	2	1	0	0
F	13	12	14	28	27	21	10	14	11	5	2	0	0
M	8	10	17	20	20	17	14	14	9	3	2	0	0
A	8	9	15	24	28	21	17	16	10	7	2	0	0
M	5	6	12	17	12	12	17	12	8	4	2	0	0
J	4	5	4	5	6	8	9	12	9	5	1	0	0
J	5	3	5	5	7	8	9	11	8	5	2	0	0
A	4	3	3	3	5	11	11	6	3	4	2	0	0
S	6	4	5	11	15	14	12	8	4	5	2	0	0
O	8	5	9	7	16	6	9	13	9	6	3	0	0
N	9	14	16	18	20	17	10	17	15	8	2	0	0
D	18	9	21	35	37	30	28	22	11	8	2	0	0

TABLE 8.—Ten-year mean values and 95 percent confidence limits of angular momentum transport by wave number six. Units: 10^{21} ergs mb. $^{-1}$

Mo.	LATITUDE ($^{\circ}$ N.)												
	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
MEANS													
J	6	16	37	46	20	12	7	-0	-1	-1	-0	+0	+0
F	9	23	36	39	17	11	15	4	-0	-0	-0	+0	+0
M	10	17	34	22	12	-1	-4	-4	-3	-1	-0	+0	+0
A	14	21	40	46	35	17	11	9	+0	-2	-0	+0	+0
M	8	15	28	45	44	31	20	11	2	-2	-1	-0	+0
J	2	7	12	16	21	21	15	5	+0	+0	+0	-0	+0
J	3	6	10	19	22	21	16	13	5	+0	+0	+0	+0
A	5	7	7	10	12	13	8	4	1	+0	-0	+0	+0
S	2	4	7	16	29	35	29	12	2	-1	-0	+0	+0
O	3	4	5	20	28	33	35	20	5	1	-0	-0	-0
N	4	15	26	41	27	16	8	-4	-5	-1	-0	-0	+0
D	9	21	42	44	20	16	16	3	-0	-1	-0	+0	+0
CONFIDENCE LIMITS													
J	19	13	18	29	27	24	14	7	5	3	1	0	0
F	27	15	20	22	21	18	13	10	5	3	1	0	0
M	11	15	22	18	18	19	14	9	5	4	2	0	0
A	10	10	12	14	17	18	9	5	4	4	1	0	0
M	8	5	7	18	25	22	16	8	5	2	0	0	0
J	4	5	7	7	8	8	10	9	5	2	0	0	0
J	5	3	6	15	14	12	9	7	5	3	1	0	0
A	3	3	4	6	4	6	8	6	4	2	1	0	0
S	5	5	5	5	6	7	14	11	8	3	1	0	0
O	5	6	9	10	19	25	19	11	6	3	1	0	0
N	13	8	11	20	17	17	14	12	6	2	1	0	0
D	18	20	18	15	26	25	17	10	7	3	1	0	0

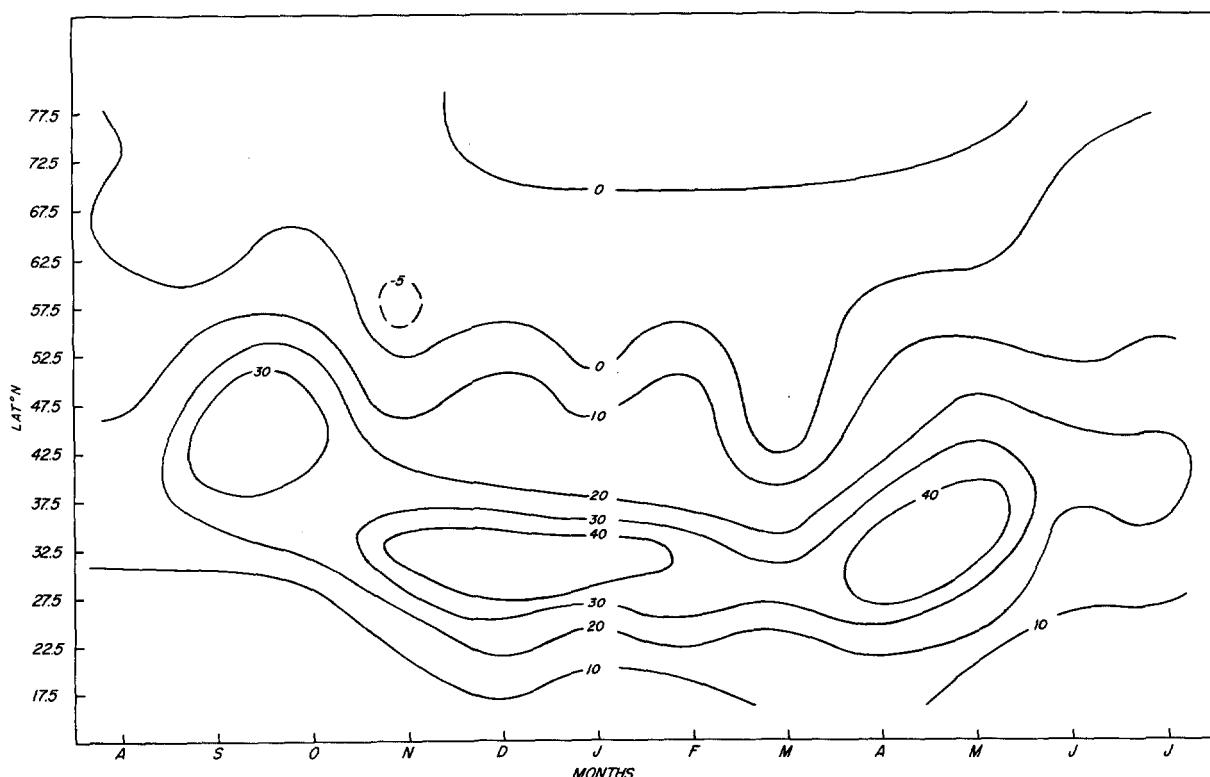
 $N=6$ 

FIGURE 4.—Same as figure 1 for wave number 6.

TABLE 9.—Ten-year mean values and 95 percent confidence limits of angular momentum transport by wave number seven. Units: 10^{21} ergs mb. $^{-1}$

Mo.	LATITUDE ($^{\circ}$ N.)												
	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
MEANS													
J	18	24	25	30	15	+0	-3	-1	-1	-1	-0	+0	+0
F	17	27	35	37	14	3	-0	-6	-3	-0	-0	+0	+0
M	13	28	43	45	22	5	-5	-4	-0	-0	+0	+0	+0
A	15	23	41	39	33	26	12	3	-1	-1	-0	-0	+0
M	6	13	22	22	24	20	15	4	-0	-0	-0	+0	+0
J	2	8	9	10	14	22	21	13	10	1	+0	-0	+0
J	+0	5	8	4	8	13	17	10	2	+0	+0	+0	+0
A	+0	3	6	3	8	13	14	6	-1	-1	-0	-0	+0
S	-0	3	5	12	21	28	25	11	4	1	+0	-0	+0
O	1	5	13	21	27	23	21	11	2	-0	+0	-0	+0
N	8	17	29	36	31	15	7	2	-0	-1	-0	-0	-0
D	17	17	27	35	30	18	2	-6	-3	-1	-0	+0	+0
CONFIDENCE LIMITS													
J	12	10	18	16	15	9	11	7	2	1	0	0	0
F	21	7	12	9	14	15	10	6	3	1	0	0	0
M	11	7	8	13	15	19	12	8	3	2	0	0	0
A	11	11	17	16	16	11	13	5	3	2	0	0	0
M	5	6	12	10	15	14	12	7	3	1	1	0	0
J	6	5	8	10	10	13	10	6	12	2	0	0	0
J	5	4	7	4	5	5	11	9	4	2	0	0	0
A	6	4	5	4	4	6	6	7	5	1	0	0	0
S	5	4	5	6	9	14	16	9	4	2	0	0	0
O	9	7	6	7	11	5	6	6	3	1	0	0	0
N	12	7	11	18	15	14	11	10	5	2	0	0	0
D	23	12	10	12	13	12	12	7	4	2	0	0	0

TABLE 10.—Ten-year mean values and 95 percent confidence limits of angular momentum transport by wave number eight. Units: 10^{21} ergs mb. $^{-1}$

Mo.	LATITUDE ($^{\circ}$ N.)												
	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
MEANS													
J	1	8	22	32	22	10	4	-0	-1	-0	-0	-0	+0
F	6	5	16	25	17	10	+0	-1	-1	-0	+0	+0	+0
M	7	6	15	20	14	6	-2	-5	-3	-0	-0	-0	+0
A	5	5	11	21	19	17	9	1	+0	+0	+0	+0	+0
M	-0	4	12	17	17	19	12	3	-0	+0	+0	+0	+0
J	1	5	10	16	15	18	11	2	-0	+0	+0	+0	+0
J	1	1	3	6	12	14	10	2	+0	-0	-0	-0	+0
A	1	2	2	5	8	10	10	4	-0	-0	+0	+0	+0
S	-0	1	8	13	18	20	12	6	1	-0	-0	-0	+0
O	1	4	10	20	19	18	12	8	1	-0	-0	-0	+0
N	9	13	16	20	21	19	13	5	+0	-0	-0	-0	+0
D	11	14	21	28	20	9	4	-2	-2	-1	-0	-0	+0
CONFIDENCE LIMITS													
J	10	10	12	18	13	10	6	3	2	1	0	0	0
F	13	16	12	14	9	11	8	6	3	1	0	0	0
M	10	8	12	9	13	14	9	4	2	1	0	0	0
A	8	6	8	10	6	7	8	5	3	2	0	0	0
M	4	5	3	6	5	9	8	5	5	3	0	0	0
J	4	3	4	6	10	11	8	3	1	0	0	0	0
J	2	2	3	4	5	6	2	2	1	0	0	0	0
A	1	3	1	4	4	2	6	4	1	0	0	0	0
S	8	5	4	6	6	4	7	6	3	1	0	0	0
O	5	3	6	7	8	7	8	9	2	0	0	0	0
N	6	5	7	12	9	10	8	4	3	1	0	0	0
D	15	9	8	11	10	8	7	4	2	1	0	0	0

TABLE 11.—Ten-year mean values and 95 percent confidence limits of angular momentum transport by wave number nine. Units: 10^{21} ergs mb. $^{-1}$

Mo.	LATITUDE ($^{\circ}$ N.)												
	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
MEANS													
J	8	9	12	14	16	6	+0	-1	-0	-0	+0	+0	+0
F	8	7	7	6	7	4	+0	-1	-1	-0	-0	+0	+0
M	8	10	6	1	1	-3	-2	-0	-0	-0	-0	-0	+0
A	6	5	10	14	13	9	12	+0	-0	-0	-0	-0	+0
M	5	7	9	15	16	14	5	+0	+0	+0	-0	-0	+0
J	1	4	9	14	12	8	2	-1	-0	-0	-0	-0	+0
J	-0	3	5	11	11	10	8	2	+0	-0	-0	-0	+0
A	+0	1	2	5	6	8	7	3	+0	+0	-0	+0	+0
S	4	2	4	10	11	9	8	3	-0	+0	+0	-0	+0
O	3	8	11	15	16	14	8	2	-0	+0	-0	-0	+0
N	5	4	6	11	12	11	6	-0	-1	-0	-0	-0	+0
D	6	9	14	16	17	9	4	-2	-2	-0	-0	+0	+0
CONFIDENCE LIMITS													
J	3	7	5	4	10	6	3	3	2	0	0	0	0
F	6	8	12	12	7	8	3	3	2	0	0	0	0
M	11	2	3	6	9	8	5	4	2	0	0	0	0
A	5	4	3	4	7	8	19	4	2	0	0	0	0
M	4	4	3	5	5	6	5	4	1	0	0	0	0
J	4	3	4	5	5	9	7	3	1	0	0	0	0
J	2	2	2	3	4	4	3	3	1	0	0	0	0
A	5	3	1	2	3	3	3	2	1	1	0	0	0
S	5	2	3	2	4	4	5	2	1	1	0	0	0
O	5	5	5	4	5	7	5	3	1	0	0	0	0
N	3	4	6	6	6	8	6	4	2	0	0	0	0
D	4	4	5	6	8	7	6	4	2	0	0	0	0

TABLE 12.—Ten-year mean values and 95 percent confidence limits of angular momentum transport by wave number ten. Units: 10^{21} ergs mb. $^{-1}$

Mo.	LATITUDE ($^{\circ}$ N.)												
	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
MEANS													
J	1	3	5	11	12	6	2	-0	-0	-0	-0	+0	+0
F	8	10	10	5	7	4	1	-2	-1	-0	-0	+0	+0
M	-0	5	6	6	+0	+0	-0	-2	-1	-0	-0	+0	+0
A	+0	5	7	9	8	6	3	-0	-0	-0	-0	+0	+0
M	+0	5	9	11	7	4	3	4	-1	-0	-0	+0	+0
J	+0	4	8	11	6	4	1	-0	+0	+0	+0	+0	+0
J	-2	3	2	4	6	6	3	1	+0	-0	+0	+0	+0
A	+0	3	2	3	4	4	3	1	-0	-0	+0	+0	+0
S	3	3	4	6	5	5	4	2	+0	+0	+0	+0	+0
O	3	6	6	8	8	4	1	-1	-0	-0	-0	+0	+0
N	1	4	7	4	5	8	2	-0	-1	-0	-0	+0	+0
D	6	8	7	7	3	1	3	1	-0	-0	-0	+0	+0
CONFIDENCE LIMITS													
J	6	2	9	9	8	6	3	2	1	0	0	0	0
F	4	4	3	7	8	7	4	2	1	0	0	0	0
M	9	3	3	5	8	6	4	2	1	0	0	0	0
A	7	4	5	6	7	5	4	1	1	0	0	0	0
M	3	1	6	7	5	4	6	9	1	0	0	0	0
J	3	3	4	5	3	2	3	2	1	0	0	0	0
J	7	1	1	2	2	2	2	2	1	1	0	0	0
A	3	3	1	2	1	2	2	1	1	0	0	0	0
S	6	3	2	3	1	3	2	2	1	1	0	0	0
O	3	3	3	2	5	5	2	3	0	0	0	0	0
N	4	3	3	3	4	3	3	2	1	0	0	0	0
D	5	3	3	3	4	3	3	2	0	0	0	0	0

TABLE 13.—Ten-year mean values, 95 percent confidence limits, and convergence ($-\Delta T_m/\Delta \phi$) of angular momentum transport by the sum of waves 1–10. Units: 10^{21} ergs mb. $^{-1}$

Mo.	LATITUDE (° N.)												
	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
MEANS													
J	45	171	315	453	432	287	111	+0	-61	-64	-44	-20	-2
F	66	184	337	457	405	286	117	-8	-60	-59	-35	-11	-1
M	57	179	302	362	325	220	73	-27	-66	-56	-31	-7	-0
A	68	133	243	313	280	202	119	9	-51	-54	-30	-8	-1
M	44	96	171	220	199	146	90	25	-34	-48	-31	-9	-0
J	11	63	97	127	149	152	104	38	-3	-20	-18	-5	-0
J	-35	10	35	49	63	78	80	59	26	-3	-8	-3	-0
A	-20	11	21	32	53	70	73	52	10	-13	-18	-8	-1
S	3	17	44	84	126	150	135	75	20	-10	-18	-7	-0
O	18	49	97	165	183	171	141	65	+0	-17	-17	-9	-3
N	48	119	205	287	287	243	153	36	-42	-46	-26	-6	+0
D	77	165	287	396	389	278	116	-15	-65	-54	-31	-9	-1
CONFIDENCE LIMITS													
J	48	36	55	67	64	69	67	57	43	26	15	10	5
F	66	64	83	103	115	121	110	88	59	29	16	8	3
M	53	58	56	62	73	68	52	30	23	18	14	7	3
A	30	28	36	37	35	25	46	35	20	11	11	7	2
M	34	20	42	46	50	42	32	27	19	12	9	5	2
J	32	17	26	24	28	27	19	15	17	13	9	5	2
J	36	13	24	31	23	14	10	12	16	14	6	2	1
A	39	18	10	13	17	12	22	20	21	13	10	5	2
S	24	13	21	23	27	24	24	14	12	6	6	4	3
O	37	17	13	24	33	28	28	29	27	19	12	6	1
N	40	26	37	26	34	41	39	37	37	27	14	5	2
D	66	38	42	50	65	69	56	45	32	27	19	10	3
CONVERGENCE													
J	-1443	-1650	-1581	240	1661	2016	1272	699	34	-229	-275	-206	...
F	-1352	-1753	-1375	595	1363	1936	1432	595	-11	-275	-275	-114	...
M	-1398	-1409	-687	424	1203	1684	1146	446	-114	-286	-275	-80	...
A	-744	-1260	-802	378	893	951	1260	687	34	-275	-252	-80	...
M	-595	-859	-561	240	607	641	744	676	160	-194	-252	-103	...
J	-595	-389	-343	-252	-34	550	756	469	194	-22	-148	-57	...
J	-515	-286	-160	-160	-171	-22	240	378	332	57	-57	-34	...
A	-355	-114	-126	-240	-194	-34	240	481	263	57	-114	-80	...
S	-160	-309	-458	-481	-275	171	687	630	343	91	-126	-80	...
O	-355	-550	-779	-208	137	343	870	744	194	0	-91	-68	...
N	-813	-985	-939	0	504	1031	1340	893	45	-229	-229	-68	...
D	-1008	-1398	-1249	80	1272	1856	1501	573	-126	-263	-252	-91	...

monthly means that are too great to permit precise location of the regions of maximum convergence of angular momentum flux. On the other hand, the general smoothness and continuity of our profiles suggest that the overall confidence is rather high. The "noise level" is sufficiently low so that the general trends are not obscured.

Keeping the above in mind, and referring again to table 13, we see that the region of transition from low-latitude divergence to mid-latitude convergence of angular momentum is generally associated with the seasonal shift of the zonal-wind maximum. The maximum convergence in winter, however, is displaced about 5° to 10° northward of the zonal wind maximum. Also, the measured eddy fluxes do not seem to account for the low-latitude summer easterlies. These features suggest that other terms in the balance equation for zonal angular momentum will ultimately have to be taken into account.

Of overall interest is the dominance of the momentum transports in winter by the larger-scale waves ($n=1-5$) and the increase in relative importance of the shorter scales of motion in summer. In fact, the role of the larger-scale waves is reduced to insignificance in July and August. Inasmuch as the momentum transport is related to the tilt of the trough and ridge lines in the atmosphere (Starr and White [10]), our results are in good agreement with the monthly mean 700-mb. trough- and ridge-line positions presented by Stark [8].

The agreement of our results for the smaller wave numbers with those of van Mieghem et al. [12], and for the sum of waves 1 through 10 with those of Starr and White [11] for the year 1950, is also worthy of note. The latter agreement is especially significant since the authors employed station data to calculate the fluxes by "standing"

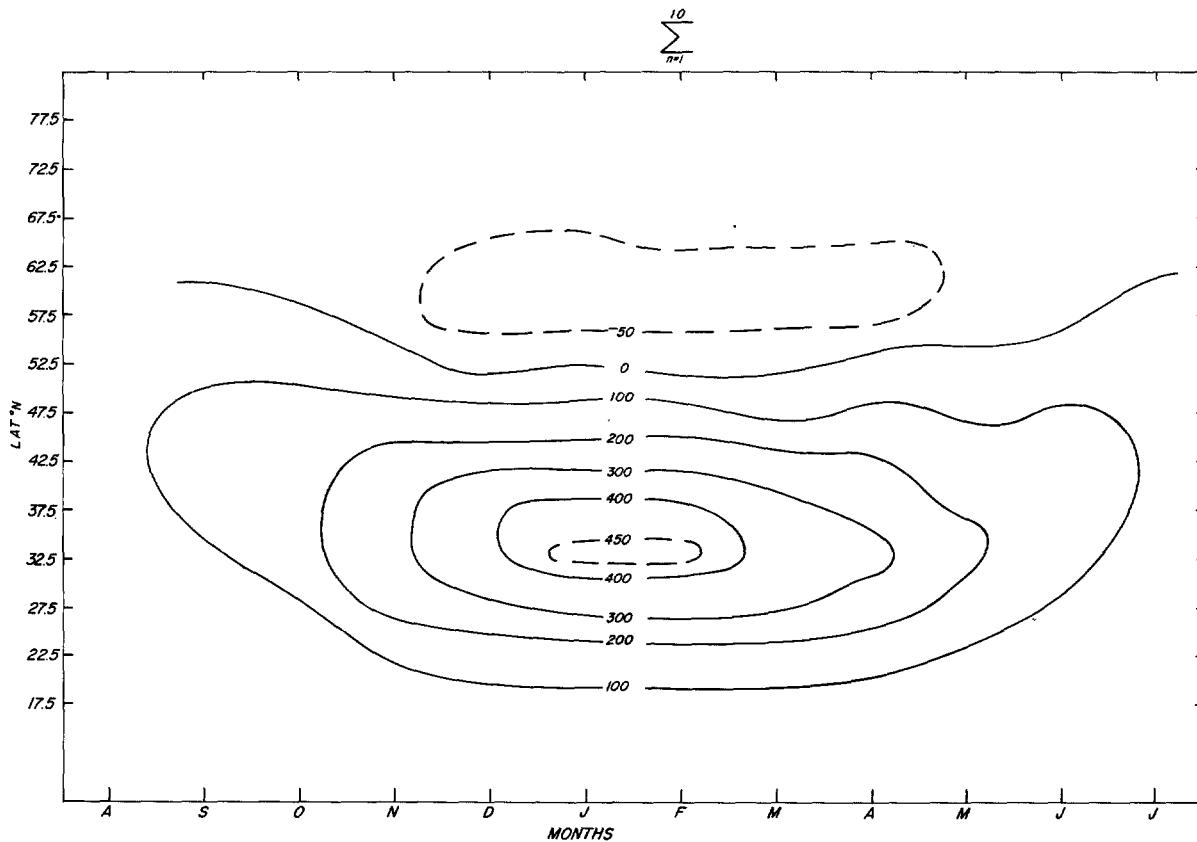


FIGURE 5.—Same as figure 1 for the sum of waves 1-10.

and "transient eddies," which were then averaged around latitude circles.

The main feature of our results, however, is the individuality that the various waves or wave-number groups exhibit in transporting angular momentum. In this respect, of particular interest are waves 2 and 3 at middle and high latitudes, and wave 2 at low latitudes in summer. This suggests that future numerical models will have to include the planetary-scale orographic and thermal effects and not concentrate solely on the medium-scale baroclinic processes.

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REFERENCES

1. H. Jeffreys, "On the Dynamics of Geostrophic Winds," *Quarterly Journal of the Royal Meteorological Society*, vol. 52, No. 217, Jan. 1926, pp. 85-104.
2. A. F. Krueger, J. S. Winston, and D. A. Haines, "Computation of Atmospheric Energy and Its Transformation for the Northern Hemisphere for a Recent Five-Year Period," *Monthly Weather Review*, vol. 93, No. 4, Apr. 1965, pp. 227-238.
3. Y. Mintz, "The Geostrophic Poleward Flux of Angular Momentum in the Month of January 1949," *Tellus*, vol. 3, No. 3, Aug. 1951, pp. 195-200.
4. B. Saltzman, "Equations Governing the Energetics of the Larger Scales of Atmospheric Turbulence in the Domain of Wave Number," *Journal of Meteorology*, vol. 14, No. 6, Dec. 1957, pp. 513-523.
5. B. Saltzman and A. Fleisher, "The Exchange of Kinetic Energy Between Larger Scales of Atmospheric Motion," *Tellus*, vol. 12, No. 4, Nov. 1960, pp. 374-377.
6. B. Saltzman and A. Fleisher, "Spectral Statistics of the Wind at 500 mb.," *Journal of the Atmospheric Sciences*, vol. 19, No. 2, March 1962, pp. 195-204.
7. B. Saltzman and S. Teweles, "Further Statistics on the Exchange of Kinetic Energy Between Harmonic Components of the Atmospheric Flow," *Tellus*, vol. 16, No. 4, Nov. 1964, pp. 432-435.

8. J. P. Stark, "Positions of Monthly Mean Troughs and Ridges in the Northern Hemisphere, 1949-1963," *Monthly Weather Review*, vol. 93, No. 11, Nov. 1965, pp. 705-720.
9. V. P. Starr, "The Physical Basis for the General Circulation," *Compendium of Meteorology*, American Meteorological Society, Boston, Mass., 1951, pp. 541-550.
10. V. P. Starr and R. M. White, "Schemes for the Study of Hemispheric Exchange Processes," *Quarterly Journal of the Royal Meteorological Society*, vol. 78, No. 337, July 1952, pp. 407-410.
11. V. P. Starr and R. M. White, "Balance Requirements of the General Circulation," *Studies of the Atmospheric General Circulation, Final Report, Part I*, Contract AF19(122)-153, Dept. of Meteorology, Massachusetts Institute of Technology, 1954, pp. 186-242.
12. J. van Mieghem, P. Defrise, and J. Van Isacker, "De Harmonische Analyse van het Normaal Windfeld van het Noordelijk Halfrond op het 500 mb. Niveau," [Harmonic Analysis of the Normal Monthly Northern-Hemisphere Geostrophic Flow at 500 mb.], *Mededelingen van de Koninklijke Vlaamsche Academie Voor Wetenschappen*, vol. 22, No. 4, 1960, 38 pp.
13. A. Wiin-Nielsen, J. Brown, and M. Drake, "Further Studies of Energy Exchange Between the Zonal Flow and the Eddies," *Tellus*, vol. 16, No. 2, May 1964, pp. 168-180.

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